A Simple Twin Prime Sieve

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October 8rd, 2010 - *draft 01*

Related to some work that I have been doing related to the Primes I have created this twin prime sieve. Mathematica code is attached at the bottom.

Twin Prime Sieve

This algorithm works for any input N which should be greater than 15. It is similar to the Euler sieve. This is not very efficient but it seems to work.

A) To find all the twin primes less than N start with all the natural numbers from 1 to 3 * ((N+2) / 3):

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 ... N

B) Remove all multiples of 2 leaving only odd numbers:

Odds: 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51...

C) Discard 3 and partition the odd numbers into groups of 3:

Odds: 5 7 9 | 11 13 15 | 17 19 21 | 23 25 27 | 29 31 33 | 35 37 39 | 41 43 45 | 47 49 51 | 53 55 57 | ...

D) Enumerate each *group* of three and *assume,* for the moment, the left two values in each *three-group* together form a twin prime. This is what will be sieved. Strike all multiples of three in Odds. Set P to the reference the first number in Odds (5).

```
          P
Odds:    5 7 _ | 11 13 _ | 17 19 _ | 23 25 _ | 29 31 _ | 35 37 _ | 41 43 _ | 47 49 _ | 53 55 _ | ...
Groups:  1       2          3          4          5          6          7          8          9          ...
```

e.g., 5, 7 are twin, 11, 13 are twin, etc. Call this set the set of *all_twin_primes*.

E) Starting with P^2 discard any *three-group* from the set of *all_twin_primes* where, for a multiple of P (call it P_m) that falls within the *three-group*, Floor[((P_m + 1)/3) / 2] identifies the *three-group* by number and Mod[P_m, 3] != 0.

So for 5 we find, starting with 25, that a multiple of 5 occurs in the *three-groups* 4, 6, 8, etc. So those *three-groups* are removed from the set *all_twin_primes*.

F) Strike all multiples of P^2 from Odds.

```
          P
Odds:    5 7 _ | 11 13 _ | 17 19 _ | 23 _  _ | 29 31 _ | _ 37 _ | 41 43 _ | 47 49 _ | 53 _ _ | ...
Groups:  1       2          3          X          5          X          7          X          9          ...
```

G) Advance P to the next number not struck in Odds.

```
            P
Odds:    5 7 _ | 11 13 _ | 17 19 _ | 23 _  _ | 29 31 _ | _ 37 _ | 41 43 _ | 47 49 _ | 53 _ _ | ...
Groups:  1       2          3          X          5          X          7          X          9          ...
```

Repeat Steps E..G while N < P^2

For each *three-group* remaining in Groups the left two values are the twin primes less than N.

Mathematica Code

```
KuenyTwinPrimeSieve[myN_]:=
 Block[{odds, groups, P, wP, primes, twins},
  odds = Table[i, {i, 5, 3*Quotient[myN +2, 3], 2}];
  groups = Table[i, {i, 1, (Quotient[myN +2, 3]/2) - 1}];DiscardFrom[table_, n_, start_] := Block[{result={}, i}, For[i = 1, i <= Length[table], i++, result = Append[result,
If[NumberQ[table[[i]]] && (table[[i]]< start || Mod[table[[i]],n] != 0), table[[i]], "_"]]]; result];
  MarkGroupsFrom[table_, group_] := Block[{i, result = {}}, For[i = 1, i <= Length[group], i++, result = Append[result, If[NumberQ[group[[i]]],
        Block[{x = group[[i]] - 1}, If[table[[x*3 + 1]] == "_" || table[[x*3 + 2]] == "_", "_", group[[i]] ]], group[[i]]]]]; result];
  P = 1;
  odds = DiscardFrom[odds, 3, 9];
  While[P < Length[odds],
   wP = odds[[P]];
   If[wP  != "_",
          Block[{},
            odds = DiscardFrom[odds, wP, wP^2];
     groups = MarkGroupsFrom[odds, groups];
     ]];
      P++;];

  twins = {};

  For[i = 1, i < Length[groups], i++, If[groups[[i]] != "_", twins = Append[twins, Block[{x = groups[[i]] - 1},{odds[[x*3 + 1]], odds[[x*3 + 2]]} ]  ]]];

  twins ]

 test1 = KuenyTwinPrimeSieve[5000]
 {{5,7},{11,13},{17,19},{29,31},{41,43},{59,61},{71,73},{101,103},{107,109},{137,139},{149,151},{179,181},{191,193},{197,199},{227,229},{239,241},{269,271},{281,283},{311,313},{347,349},
{419,421},{431,433},{461,463},{521,523},{569,571},{599,601},{617,619},{641,643},{659,661},{809,811},{821,823},{827,829},{857,859},{881,883},{1019,1021},{1031,1033},{1049,1051},{1061,1063},
{1091,1093},{1151,1153},{1229,1231},{1277,1279},{1289,1291},{1301,1303},{1319,1321},{1427,1429},{1451,1453},{1481,1483},{1487,1489},{1607,1609},{1619,1621},{1667,1669},{1697,1699},
{1721,1723},{1787,1789},{1871,1873},{1877,1879},{1931,1933},{1949,1951},{1997,1999},{2027,2029},{2081,2083},{2087,2089},{2111,2113},{2129,2131},{2141,2143},{2237,2239},{2267,2269},
{2309,2311},{2339,2341},{2381,2383},{2549,2551},{2591,2593},{2657,2659},{2687,2689},{2711,2713},{2729,2731},{2789,2791},{2801,2803},{2969,2971},{2999,3001},{3119,3121},{3167,3169},
{3251,3253},{3257,3259},{3299,3301},{3329,3331},{3359,3361},{3371,3373},{3389,3391},{3461,3463},{3467,3469},{3527,3529},{3539,3541},{3557,3559},{3581,3583},{3671,3673},{3767,3769},
{3821,3823},{3851,3853},{3917,3919},{3929,3931},{4001,4003},{4019,4021},{4049,4051},{4091,4093},{4127,4129},{4157,4159},{4217,4219},{4229,4231},{4241,4243},{4259,4261},{4271,4273},
{4337,4339},{4421,4423},{4481,4483},{4517,4519},{4547,4549},{4637,4639},{4649,4651},{4721,4723},{4787,4789},{4799,4801},{4931,4933},{4967,4969}}
 Map[First, test1]

{5,11,17,29,41,59,71,101,107,137,149,179,191,197,227,239,269,281,311,347,419,431,461,521,569,599,617,641,659,809,821,827,857,881,1019,1031,1049,1061,1091,1151,1229,1277,1289,1301,1319,1427,14
51,1481,1487,1607,1619,1667,1697,1721,1787,1871,1877,1931,1949,1997,2027,2081,2087,2111,2129,2141,2237,2267,2309,2339,2381,2549,2591,2657,2687,2711,2729,2789,2801,2969,2999,3119,3167,3251,325
7,3299,3329,3359,3371,3389,3461,3467,3527,3539,3557,3581,3671,3767,3821,3851,3917,3929,4001,4019,4049,4091,4127,4157,4217,4229,4241,4259,4271,4337,4421,4481,4517,4547,4637,4649,4721,4787,4799
,4931,4967}
```